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PRODUCTION OF γ-RAYS IN THE CRAB NEBULA PULSAR

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Summary

The γ-ray flux at energies above 50 MeV from NP0532 is evaluated for a Pulsar model in which the optical and X-radiation is produced by synchrotron effect. Synchrotron radiation and inverse Compton scattering are considered as production mechanisms of the γ-rays. The theoretical estimates are compared with the experimental values.

Riassunto

Si calcola il flusso di raggi γ di energia maggiore di 50 MeV da NP0532 usando un modello di Pulsar nel quale la radiazione ottica e la radiazione X sono prodotte per effetto di sincrotrone. Si considerano come meccanismi di produzione dei raggi γ, la radiazione di sincrotrone e la diffusione Compton inversa. Le previsioni teoriche sono confrontate con i valori sperimentali.

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1. Introduction

Recently Vasseur et al.[1] have reported evidence that NP0532, the Pulsar in the Crab Nebula, is pulsating at γ-ray frequencies. The pulsed flux at energies above 50 MeV was found to be of the order of $10^{-5}\gamma/cm^2 sec$, which corresponds to an energy output of $\sim 10^{35}$ erg/sec.

In this paper the γ-ray observations are related to a model of the Pulsar in which the optical and X-radiations are the synchrotron radiation of relativistic electrons moving around the lines of force of the magnetic field.

A model of this kind has been discussed by Shklovsky[2]. From the similarity of the spectral shape of the Pulsar in the X-ray domain and of the Nebula in the radio domain, he deduces a value $H_\perp \sim 4x10^3$ gauss for the perpendicular component of the magnetic field in the emitting region of the Pulsar. By the same argument he suggests that the electron spectrum $N(E)=\kappa E^{-\alpha}$ has a slope $\alpha=1.6$ for energies $E_e \leq 5x10^9$eV; $\alpha=2.4$ for $E_e \geq 5x10^9$eV. Assuming that the emitting region has a linear dimension $\ell \sim 10^7$cm from the observed X-ray flux, the following values of the spectral index κ are obtained:

$$\kappa = 5x10^{11} erg^{\alpha-1} cm^{-3} \quad \text{for } E_e \leq 5x10^9 eV;$$
$$\kappa = 10^{10} erg^{\alpha-1} cm^{-3} \quad \text{for } E_e \geq 5x10^9 eV.$$

In the following we shall use these parameters to give a quantitative estimate of the γ-ray fluxes.

In section 2 the contribution to the γ-ray flux due to synchrotron effect is evaluated.

In section 3 the contribution due to the Compton scattering of relativistic electrons colliding with optical and X-ray photons (Compton-synchrotron effect) is examined; the γ-ray flux is given as a function of the scattering angle because the photon and electron distributions can be highly anisotropic. The relevant formulae for inverse Compton scattering at small angles are given in Appendix I.

A discussion of the results in given in section 4.

2. Synchrotron Radiation

Synchrotron radiation is an obvious process for explaining the γ-ray observations of Vasseur et al. because the observed flux lies near the extrapolation of the X-ray spectrum. From fig. 2 of Fishman et al.[3] the following extrapolated value for the pulsed flux above 50 Mev is obtained:

$$I_\gamma(>50\ \mathrm{MeV}) \simeq 4\times10^{-6}\gamma/\mathrm{cm}^2\mathrm{sec}.$$

In the Shklovsky model the contribution due to synchrotron radiation can be even greater because the ratio of the pulsed component to the steady component is supposedly increasing with increasing energy so that above 50 Mev the flux can be entirely pulsed. The well known formula for synchrotron radiation (Ginzburg and Syrovatskii[4]) reads:

$$I_\nu \times \nu = \frac{1.35\times10^{-22} a(\alpha)\kappa V H_\perp^{\frac{\alpha+1}{2}} (6.6\times10^{18})^{\frac{\alpha-1}{2}} \nu^{\frac{3-\alpha}{2}}}{R^2} \qquad (1)$$

(c.g.s. units)

where $a(\alpha)\simeq8.5\times10^{-2}$ for $\alpha=2.4$; V is the volume of the emitting region, $\sim10^{21}\mathrm{cm}^3$; R is the distance of the source, $\sim3\times10^{21}$cm and ν is the frequency. We have then:

$$I_\gamma(>50\ \mathrm{MeV}) \simeq 10^{-}\ \mathrm{erg/cm^2 sec} \simeq 10^{-5}\gamma/\mathrm{cm^2 sec} \qquad (2)$$

The energy of the electrons producing the γ-ray photons is:

$$E_e = 7.6\mathrm{x}10^9 \left[\frac{E_\gamma(\mathrm{eV})}{H_\perp(\mathrm{gauss})}\right]^{1/2} \mathrm{eV}, \qquad (3)$$

for $E_\gamma > 50$ MeV it is:

$$E_e \gtrsim 10^{12}\mathrm{eV}.$$

3. Inverse Compton Scattering

In the model considered the photons are produced by relativistic electrons by synchrotron effect. They must then have the same direction as do the electrons. These electrons are supposed to move on trajectories characterized by small pitch angles ϕ. Shklovsky suggests $\phi \simeq 10^{-3}$, a value which has been criticized by O'Dell and Sartori[5] on the basis of the theory of synchrotron radiation from an anisotropic distribution of electrons. The condition of pulsation[5] gives:

$$\phi < \frac{\tau}{P} \simeq 10^{-1}$$

where τ is the pulse width and P the period. If the magnetic field were completely uniform, the scattering angle θ between electrons and photons would be $\simeq \phi$. The magnetic field in the region of emission at the moment is unknown. Nevertheless, it seems reasonable that the effective scattering angle θ_{eff} be small, although probably greater than ϕ.

The photon spectrum arising from Compton scattering at small angles is given by formula (A-10).

$$I_\gamma \simeq \theta_{eff}^{\alpha+1}\, I_{\gamma\ \mathrm{isotropic}} \qquad (4)$$

and (Ginzburg and Syrovatskii[6])

$$I_\gamma(E_\gamma)_{\text{isotropic}} = 5.5\times10^{-22} \frac{W}{\bar{E}_{ph}} \frac{V}{R^2} \frac{1}{\sqrt{\bar{E}_{ph}E_\gamma}} \kappa_e \bar{E}_e^{-\alpha} \ \gamma/\text{cm}^2\text{sec erg}$$

where $\bar{E}_{ph}$ is the mean energy of the X-ray photons, 2×10^{-8}erg; W is the photon energy density, 6×10^{10}erg/cm^3 as deduced by the X-ray observations, and

$$\bar{E}_e \sim \sqrt{\frac{3}{4}\frac{E_\gamma}{\bar{E}_{ph}}}\ mc^2 .$$

For $E_\gamma>50$ MeV, it is:

$$I_\gamma(>50\text{ MeV})_{\text{isotropic}} \sim 3\times10^{-6}\ \gamma/\text{cm}^2\text{sec}$$

The energy of the electrons producing the γ-ray photons is given by (A-5):

$$E_e \sim \sqrt{\frac{2E_\gamma}{\bar{E}_{ph}}}\ mc^2\ \theta^{-1} \tag{5}$$

For $E_\gamma \simeq 50$ MeV

$$E_e \sim 50\ \theta^{-1}\text{MeV}$$

The mean life τ_c of electrons for Compton losses is given by:

$$\tau_c = E/\frac{dE}{dt}$$

where $\frac{dE}{dt}$ is the electron energy loss for Compton scattering

$$\frac{dE}{dt} \sim \theta^4 \ \frac{dE}{dt}_{\text{isotropic}} \quad \text{(see formula A-9)}$$

and $$\frac{dE}{dt} \simeq 10^{-25} WE^2 \text{ ev/sec}$$

where W is measured in eV/cm^3, E in eV. We have then:

$$\tau_c \sim \frac{10^{25}\theta^{-4}}{WE} \text{ sec} \tag{6}$$

4. Discussion

In table 1 the results of the previous calculations are summarized. The first column gives the flux of γ-rays of energy greater than 50 MeV; the contribution of Compton scattering is calculated from formula (4) for different scattering angles θ_{eff}. In the second column the energy of the electrons necessary for production of γ-rays of 50 MeV is evaluated using formulae (3) and (5); the mean life for Compton losses of these electrons, obtained by formula (6), is reported in the third column.

Some consequences can be deduced from the above values. If the electron spectrum maintains the slope $\alpha=2.4$ up to 10^{12}eV, synchrotron radiation is the dominant effect in producing γ-rays above 50 MeV. Compton scattering is an important effect only if the interaction between electrons and photons can be considered isotropic, i.e., $\theta_{eff} \sim 1$. In this case the energy required for the electrons to produce γ-rays would be $E_e \sim 100$ MeV. We observe that an isotropic distribution of electrons implies a very short life for Compton losses. For instance, the electrons which produce X-rays of 10 keV, whose energy is $E_e \sim 10^{10}$eV, would live only for 10^{-8} sec in comparison with the pulse width $\tau \simeq 10^{-6}$ sec.

The mechanisms examined for producing γ-rays, i.e., synchrotron radiation and Compton scattering, will be active even if the structure of the source is different from that of the model we have considered. Nevertheless, it is obvious that, were the values of the physical parameters much different from those proposed by Shklovsky, the estimates of the γ-ray flux should be changed. On the other hand, if new experiments confirm the existence of a γ-ray flux $I_{\gamma} \sim 10^{-5} - 10^{-6} \gamma/cm^2 sec$ above 50 MeV, an indirect test for the model will be achieved.

Appendix - Inverse Compton Scattering at Small Angles

a. General Formulae

Baylis, Schmidt and Lusher[7] (B.S.L. in the following text) have studied inverse Compton effect for these scattering angles: $\theta=0$; $\theta=\frac{\pi}{2}$; $\theta=\pi$. We are interested in the case $0<\theta<<1$. The treatment by B.S.L. will be followed closely.

Let (ω,κ), (ω',κ') be the energy and momentum of the photons before and after the collision; (γ,p), (γ',p'), the energy and momentum of the electrons; χ, χ', the angles defined by:

$$\chi \simeq \kappa\hat{p}, \quad \chi' \simeq \kappa'\hat{p}; \quad \gamma=(1-v^2)^{-1/2}; \quad r_e = 2.82\times10^{-13}\,\mathrm{cm}$$

We put $h = c = m_e = 1$.

The scattering differential cross section $\frac{d\sigma}{d\omega'}$ can be obtained by the Klein Nishina formula by application of a Lorentz transformation. For ω, ω', $1 << \gamma$, it is (B.S.L., formula 24):

$$\frac{d\sigma}{d\omega'} = \frac{2\pi r_e^2}{\gamma\kappa}\left[1 - \frac{\omega+\omega'}{\gamma\kappa} + \frac{\omega^2+6\omega\omega'+\omega'^2}{2\gamma^2\kappa^2} - \frac{3\omega\omega'(\omega+\omega')}{3\gamma^2\kappa^3} + \frac{3\omega^2\omega'^2}{4\gamma^4\kappa^4}\right]$$

where $\kappa = \gamma\omega(1-v\cos\chi)$.

The electron energy loss per unit time is given by:

$$\frac{d\gamma}{dt} = -\int d\omega \int d\omega' \int d\Omega\,(\omega'-\omega)\,\frac{d\sigma}{d\omega'}\frac{\partial^2 n}{\partial\Omega\,\partial\omega}(1-v\cos\chi) \qquad \text{(A-1)}$$

and the spectrum of the created photons by:

$$\frac{dR}{d\omega'} = \int d\omega \int d\gamma \int d\Omega\,\frac{dF}{d\gamma}\,\frac{d\sigma}{d\omega'}\frac{\partial^2 n}{\partial\Omega\,\partial\omega}(1-v\cos\chi) \qquad \text{(A-2)}$$

where $\frac{dF}{d\gamma}$ is the spectrum of the electrons and $\frac{\partial^2 n}{\partial\Omega\partial\omega}$, the spectrum of the ambient photons per unit solid angle. For a discussion of formulae (A-1) and (A-2), we refer to B.S.L., section IV.

We suppose that the shape of the spectrum of the ambient photons be independent of the direction, i.e.,

$$\frac{\partial^2 n}{\partial\Omega\partial\omega} = \frac{dn(\omega)}{d\omega}\,\Psi(\Omega).$$

We can then define a function

$$I(\omega,\omega',\gamma) = \int d\Omega \frac{d\sigma}{d\omega'}\Psi(\Omega)(1-v\cos\chi)$$

so that (A-1) and (A-2) can be written in the form:

$$\frac{d\gamma}{dt} = -\iint d\omega d\omega' \frac{dn}{d\omega} I(\omega',\omega,\gamma)(\omega-\omega') \tag{A-3}$$

$$\frac{dR}{d\omega'} = \iint d\gamma d\omega \frac{dF}{d\gamma}\frac{dn}{d\omega}\, I(\omega',\omega,\gamma) \tag{A-4}$$

For the case of interest to us the integrals (A-3) and (A-4) must be calculated for:

$$\Psi(\Omega) = \frac{1}{2\pi}\,\delta(\cos\chi - \cos\theta)$$
$$\simeq \frac{1}{2\pi}\,\delta(\cos\chi - 1 + \frac{\theta^2}{2}) \quad \text{for } \theta \ll 1$$

and correspondingly

$$I(\omega,\omega',\gamma) = \frac{2\pi r_e^2}{\gamma^2\omega}\,[1 - \frac{\theta}{2} - \frac{2}{\gamma^2\theta^2} - \frac{2}{\gamma^2\theta^2}\frac{\omega'}{\omega} + \frac{2}{\gamma^4\theta^4}$$
$$+ 12\frac{\omega'}{\omega}\frac{1}{\gamma^4\theta^4} + 2\frac{\omega'^2}{\omega^2}\frac{1}{\gamma^4\theta^4} - 12\frac{\omega'}{\omega}\frac{1}{\gamma^6\theta^6}$$
$$- 12\frac{\omega'^2}{\omega^2}\frac{1}{\gamma^6\theta^6} + 12\frac{\omega'^2}{\omega^2}\frac{1}{\gamma^8\theta^8}].$$

The range of variation of ω' is a complicated function of χ, χ', γ, ω (see Felten and Morrison[8], formula 5); if the supplementary hypothesis $\gamma\omega<<1$ is made, we obtain:

$$\gamma^2\omega(1 - \beta\cos\theta)(1 - \beta) < \omega' < 2\gamma^2\omega(1 - v\cos\theta)$$

and for $\theta<<1$,

$$\frac{\theta^2}{4}\omega < \omega' < \gamma^2\theta^2\omega. \tag{A-5}$$

b. The Energy Loss of the Electrons

For $\theta<<1$ we have:

$$\int_{\frac{\theta^2}{4}\omega}^{\gamma^2\theta^2\omega} I(\omega,\omega',\gamma)\,d\omega' = \frac{4}{3}\pi r_e^2\theta^2 ,$$

$$\int_{\frac{\theta^2\omega}{4}}^{\gamma^2\theta^2\omega} I(\omega,\omega',\gamma)\,\omega'd\omega' = \frac{2}{3}\pi r_e^2\gamma^2\theta^4\omega ,$$

and hence:

$$\frac{d\gamma}{dt} = -\frac{2}{3}\pi r_e^2(\gamma^2\theta^4 - 2\theta^2)\int\omega\frac{dn}{d\omega}d\omega \tag{A-6}$$

which indicates that the electrons will gain energy when $\gamma^2\theta^2<2$ and lose energy in the opposite case. For $\gamma^2\theta^2>>2$:

$$\frac{d\gamma}{dt} = -\frac{2}{3}\pi r_e^2\gamma^2\theta^4\int\omega\frac{dn}{d\omega}d\omega \tag{A-7}$$

which is the same expression given by Woltjer[9].

c. The Photon Spectrum

Let the electron spectrum be:

$$\frac{dF}{d\gamma} = \kappa_e \gamma^{-\alpha} \qquad \gamma_1 < \gamma < \gamma_2$$

and the ambient photon spectrum:

$$\frac{dn}{d\omega} = \kappa_\omega \omega^{-\beta} \qquad \omega_1 < \omega < \omega_2 .$$

The integration of (A-4) gives the following:

$$\frac{dR}{d\omega'} = \int_{(\omega'/\theta^2\omega)^{1/2}}^{\infty} d\gamma \frac{dF}{d\gamma} \frac{\partial n}{\partial \omega} I(\omega,\omega',\gamma)$$

$$= 2\pi r_e^2 \kappa_\omega \kappa_e \theta^{\alpha+1} \omega'^{\frac{-\alpha-1}{2}} \omega^{\frac{\alpha+1-2\beta}{2}} \times \Bigg\{ \Big[\frac{2}{\alpha+1-2\beta} f_1(\alpha) + \frac{2}{\alpha+3-2\beta}\left(\frac{\omega}{\omega'}\right) f_2(\alpha) + \frac{2}{\alpha+5-2\beta}\left(\frac{\omega}{\omega'}\right)^2 f_3(\alpha)\Big] - \frac{\theta^2}{(\alpha+1)(\alpha+1-2\beta)}\Bigg\}\Bigg|_{\omega_1}^{\omega_2}$$

where

$$f_1(\alpha) = \frac{1}{\alpha+1} - \frac{2}{\alpha+3} + \frac{2}{\alpha+5}$$

$$f_2(\alpha) = \frac{2}{\alpha+3} - \frac{12}{\alpha+5} + \frac{12}{\alpha+7}$$

$$f_3(\alpha) = \frac{2}{\alpha+5} - \frac{12}{\alpha+7} + \frac{12}{\alpha+9}$$

For $\theta \ll 1$ and $\omega' \gg \omega_2$

$$\frac{dR}{d\omega'} = \frac{4\pi r_e^2 \kappa_\omega \kappa_e}{\alpha+1-2\beta} \theta^{\alpha+1} \omega'^{\frac{-\alpha-1}{2}} f_1(\alpha) \left[\omega_2^{\frac{\alpha+1-2\beta}{2}} - \omega_1^{\frac{\alpha+1-2\beta}{2}}\right].$$

Expressions (A-6) and (A-8) indicate that the energy loss per unit time, $\frac{d\gamma}{dt}$, depends on the fourth power of the scattering angle, θ, and the photon spectrum, $\frac{dR}{d\omega'}$, on $\theta^{\alpha+1}$.

Disregarding small numerical factors we can then write:

$$\frac{d\gamma}{dt} \sim \theta^4 \left(\frac{d\gamma}{dt}\right)_{\text{isotropic}} \qquad \text{(A-9)}$$

$$\frac{dR}{d\omega'} \sim \theta^{\alpha+1} \left(\frac{dR}{d\omega'}\right)_{\text{isotropic}} \qquad \text{(A-10)}$$

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Table 1

I_γ(>50 MeV) is the flux of γ-rays above 50 MeV measured at Earth; E_e, the energy of the electrons which produce photons of 50 MeV; τ_c, the mean life of the electrons for inverse Compton scattering.

	I_γ(>50 MeV) γ/cm^2 sec	E_e (eV)	τ_c (sec)
measured flux	$\sim 10^{-5}$		
extrapolation from X-ray observations	$\sim 4 \times 10^{-6}$		
synchrotron $\alpha = 2.4$	$\sim 10^{-5}$	$\sim 10^{12}$	[isotropic] $\sim 10^{-10}$ [$\theta_{eff} \sim 10^{-1}$] $\sim 10^{-6}$ [$\theta_{eff} \sim 10^{-3}$] $\sim 10^{2}$
Compton isotropic	$\sim 3 \times 10^{-6}$	$\sim 10^{8}$	$\sim 10^{-6}$
Compton $\theta_{eff} \sim 10^{-1}$	$\sim 1.2 \times 10^{-8}$	$\sim 10^{9}$	$\sim 10^{-2}$
Compton $\theta_{eff} \sim 10^{-3}$	$\sim 5 \times 10^{-14}$	$\sim 10^{11}$	$\sim 10^{6}$

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